UMEÅ UNIVERSITY

Advanced Techniques in Probabilistic Combinatorics

(Avancerad teknik i probabilistisk kombinatorik)

Credit: 10 ECTS (consists of four independent 2.5 ECTS modules)

Course coordinator:
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Course Period:
June – December 2024

Main field of study and progress level:
Mathematics, PhD

Prerequisites:
Students should possess mathematical maturity, a solid background in combinatorics, probability theory, and graph theory, and some previous exposure to probabilistic combinatorics.

Objective
This course will cover four advanced probabilistic techniques typically not covered in a first course in probabilistic combinatorics: entropy, dependent random choice, the differential equation method, and pseudo-randomness.

Contents:
The probabilistic method, originating in the work of Erdős in the 1940s, has become one of the most fruitful and powerful approaches to problems in combinatorics, number theory and computer science. The fundamental insight behind the method is that for many problems, random choices or processes may be the best way to prove deterministic bounds or construct optimal structures.

A first course in probabilistic combinatorics typically covers the first and second-moment methods, the local lemma and a few results on concentration of measure. This course aims to explore four more advanced techniques that have had spectacular applications to many major problems in extremal combinatorics.

Entropy lay at the heart of Gilmer's breakthrough on the union-closed sets conjecture, among other things. Dependent random choice was introduced by Gowers in his landmark proof of Szemerédi's theorem and has had numerous powerful applications to the study of Turán numbers for bipartite graphs. The differential equation method has, amongst other things, yielded the best lower bounds for the Ramsey number R(3,t) via an analysis of the triangle-free process due to Bohman-Keevash and Fiz Pontiveros-Griffiths-Morris. Pseudo-randomness has been an immensely fruitful concept since its introduction in the work of Thomason, Chung and Graham. It has informed the proof of several landmark extremal results in recent years.

In this course, the student will familiarise themselves with all four techniques, learn to apply them, and recognise their strengths and limitations.

Form of instruction:
The teaching methods are self-study combined with scheduled meetings to discuss course content. The primary reading materials for the course are the papers listed under the literature section, together with Alon and Spencer's *The Probabilistic Method* as a reference.

Examination:
The examination consists of a series of oral presentations, one for each of the four techniques of the course.

Literature:
The primary course literature will be

1. Galvin, David. "Three tutorial lectures on entropy and counting." *arXiv preprint arXiv:1406.7872* (2014).
2. Chase, Zachary, and Shachar Lovett. "Approximate union closed conjecture." *arXiv preprint arXiv:2211.11689* (2022).
3. Gilmer, Justin. "A constant lower bound for the union-closed sets conjecture." *arXiv preprint arXiv:2211.09055* (2022).
4. Balister, Paul, and Béla Bollobás. "Projections, entropy and sumsets." *Combinatorica* 32.2 (2012): 125-141.
5. Fox, Jacob, and Benny Sudakov. "Dependent random choice." *Random Structures & Algorithms* 38.1-2 (2011): 68-99.
6. Wormald, Nicholas C. "The differential equation method for random graph processes and greedy algorithms." *Lectures on approximation and randomized algorithms* 73.155 (1999): 0943-05073.
7. Krivelevich, Michael, and Benny Sudakov. "Pseudo-random graphs." More sets, graphs and numbers: A Salute to Vera Sos and András Hajnal. Berlin, Heidelberg: Springer Berlin Heidelberg, 2006. 199-262.
8. Alon, Noga, and Joel H. Spencer. *The probabilistic method*. John Wiley & Sons, 2016.