Course Syllabus
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Spring 2025

UMEÅ UNIVERSITY

Optimal Transport

Credits: 7.5 ECTS
Course Period: March 3 – April 25, 2025
Main field of study: Mathematics/Mathematical Statistics/Computer Science
Progress Level: PhD
Grading scale: Pass/fail
Teacher: Jakob Hultgren

Description

Optimal transport is at the intersection of mathematics, mathematical statistics and computer science. At its core lies a natural way to measure the distance between probability distributions or histograms. This simple tool has been used to solve problems in geometry, machine learning, statistics, partial differential equations, economics, graph theory and mathematical biology.

Objective

The objective of the course is for the participants to

- reach a solid understanding of the optimal transport problem, both its continuous and discrete formulations, including their respective duals and qualitative descriptions of minimizers.
- get an overview of numerical methods, in particular entropic regularization and the Sinkhorn algorithm.
- experience the breadth of the topic in terms of applications.

Prerequisites

We will do our best to make the material accessible to anybody interested in taking the course. Some of the lectures will go deeper into topics and require some specialized knowledge (for example measure theory, partial differential equations or familiarity with topics in statistics or machine learning), but the bulk of the material will be accessible to any PhD student in mathematics and mathematical statistics and any PhD student in computer science with a strong interest in mathematics.

Course Structure and Examination

The course will run over eight weeks, and we will meet twice a week, perhaps a bit less towards the end of the course. The first four weeks of the course will consist of lectures and exercise sessions, covering the basics of the subject. The following four weeks will be student presentations on applications across various fields. These presentations will be the basis of examination and participation on these presentations is mandatory for all students taking the course for credits.

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Litterature

We will use two books as main references, one theoretical and one computational, both available in preprint form at arXiv and HAL, respectively.

A Users Guide to Optimal Transport
Luigi Ambrosio and Nicola Gigli.
Lecture Notes in Mathematics, vol 2062. Springer
(https://hal.science/hal-00769391v1/document)

Computational Optimal Transport
Gabriel Peyré and Marco Cuturi
Foundations and Trends in Machine Learning, vol. 11, no. 5-6, pp. 355-607, 2019
(https://arxiv.org/abs/1803.00567)

Content

The first part of the course will cover

- Continuous and discrete optimal transport
- Primal and dual formulations
- Convex Functions and Legendre Transform
- The Knott-Smith Optimality Criterion and Monge-Ampère equations
- Entropic Regularization and Sinkhorn's algorithm
- Wasserstein Space as a metric and geodesic space

The second part of the course will be focused on applications. Many of these will be outside of the organizer's expertise, so the topics will be chosen based on students' interest and how they want to contribute. Some examples of topics are:

Statistics and machine learning:
Wasserstein barycenters and clustering
(https://proceedings.mlr.press/v32/cuturi14.html & https://doi.org/10.1016/j.eswa.2013.12.001)

Differential geometry:
Synthetic Ricci curvature bounds
(https://doi.org/10.4007/annals.2009.169.903)

Partial differential equations:
Diffusion equations as gradient flows on Wasserstein space
(https://doi.org/10.1081/PDE-100002243)

Mathematical biology:
Cell perturbation response
(https://www.nature.com/articles/s41592-023-01969-x)

Graph Theory:
Graph comparisons
https://openreview.net/pdf?id=ByxlnrBeLH

Forward Kinematics:
(https://sites.google.com/view/sinkhorn-step/ and https://arxiv.org/abs/2309.15970)