UMEÅ UNIVERSITY

Erdős-Ko-Rado combinatorics of strongly regular graphs

(Erdős-Ko-Rado kombinatorik av starkt reguljära grafer)

Credit: 5.0 ECTS

Course coordinator:
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Course Period: January-February 2025

Main field of study and progress level: *Mathematics*, PhD

Grading scale: Pass/Fail

Prerequisites:
Students should have a good background in linear algebra and finite fields, and a basic understanding of graph theory and combinatorics.

Objective
This course introduces the classical Erdős-Ko-Rado theorem and its extensions to strongly regular graphs. The course is intended to be accessible to graduate students in mathematics and mathematically proficient students in algebra and discrete mathematics.

Contents:
This course initially covers the classical Erdős-Ko-Rado (EKR) theorem and an alternative approach to it, involving a tool from algebraic graph theory called Hoffman bound. Further, the course introduces strongly regular graphs, one of the central objects in algebraic graph theory. Finally, the course covers possible extensions of the EKR theorem to strongly regular graphs, namely block graphs of orthogonal arrays and 2-designs.

Plan of lectures:
1) Erdős-Ko-Rado (EKR) theorem and an alternative approach to it.
2) Strongly regular graphs and their EKR properties.
3) EKR theorem for Paley graphs of square order.
4) Possible analogue of the Hilton-Milner theorem for Paley graphs of square order.
5) EKR properties of the collinearity graphs of Desarguesian nets.
6) EKR properties of 2-designs.

Form of instruction:
The teaching consists of lectures and discussion sessions involving a demonstration of using the computer algebra system MAGMA.

Examination:
The examination consists of compulsory assignments for which written reports and complete solutions (theoretical and computational) are to be submitted.

Literature:

[1] C. Godsil, K. Meagher, *Erdős–Ko–Rado Theorems: Algebraic Approaches*, Cambridge University Press, 2015.
[2] C. Godsil, G. F. Royle, *Algebraic Graph Theory*, Graduate Texts in Mathematics, 207, 2001.
[3] A. E. Brouwer, H. Van Maldeghem, *Strongly Regular Graphs*, Cambridge University Press, 2022.
[4] A. Blokhuis, On subsets of GF(q^2) with square differences, *Indag. Math.* 46 (1984) 369–372.
[5] S. Asgarli, C.H. Yip, Van Lint-MacWilliams' conjecture and maximum cliques in Cayley graphs over finite fields, *J. Comb. Theory*, Ser. A 192 (2022) 105667
[6] S. Goryainov, C. H. Yip, Extremal Peisert-type graphs without the strict-EKR property, *Journal of Combinatorial Theory, Series A*, Volume 206, August 2024, 105887.
[7] S. Goryainov, E. V. Konstantinova, Non-canonical maximum cliques without a design structure in the block graphs of 2-designs, *Designs, Codes and Cryptography*, 92, 3665–3675 (2024).
[8] R. D. Baker, G. L. Ebert, J. Hemmeter, A. J. Woldar, Maximal cliques in the Paley graph of square order, *J. Statist. Plann. Inference* 56 (1996), 33–38.
[9] S. V. Goryainov, V. V. Kabanov, L. V. Shalaginov, A. A. Valyuzhenich, On eigenfunctions and maximal cliques of Paley graphs of square order, *Finite Fields and Their Applications*, Volume 52, 361–369, 2018.
[10] S. Goryainov, A. Masley, L. Shalaginov, On a correspondence between maximal cliques in Paley graphs of square order, *Discrete Mathematics*, Volume 345, Issue 6, June 2022, 112853.