Solution 1. Let n be an integer. Note that for any $x \in \mathbb{R}$, we have $\lfloor x+n \rfloor = \lfloor x \rfloor + n$. Indeed, this standard property follows from the fact that as $m = \lfloor x \rfloor$ is the unique integer chosen so that $m \leq x < m+1$, we also have $m+n \leq x+n < m+n+1$, thus implying that $m+n$ is the unique such integer for $x+n$.

Using this observation, we note that $\{x+n\} = \{x\}$ for any $x \in \mathbb{R}$ and $n \in \mathbb{Z}$, and hence we also have $\{(x+n)+(y+m)\} = \{x+y\}$ and $\{x+n\} \cdot \{y+m\} = \{x\}\{y\}$. Thus (x,y) is a solution for the equation if and only if $(x+n, y+m)$ is a solution for the equation for all integers $n, m \in \mathbb{Z}$. In particular, by choosing $n = -\lfloor x \rfloor$ and $m = -\lfloor y \rfloor$, it suffices to find those solutions (x,y) with $0 \leq x < 1$ and $0 \leq y < 1$. The advantage is that for these choices of x and y we have $\lfloor x \rfloor = 0$, so $\{x\}$ is simply x, while $0 \leq x+y < 2$, so $\{x+y\}$ is either $x+y$ or $x+y-1$.

First suppose that $(x,y) \in [0,1) \times [0,1)$ is a solution for which we have $0 \leq x+y < 1$. Then the original equation $\{x+y\} = \{x\} \cdot \{y\}$ can be written as $x+y = xy$, which again can be written as $(1-x)(1-y) = 1$. Since $x, y \in [0,1)$, we have $0 < 1-x \leq 1$ and $0 < 1-y \leq 1$, which implies that $(1-x)(1-y) \leq 1$. Since the equality must hold, we need both $1-x$ and $1-y$ to be 1, that is we need $x = y = 0$. Thus the only solution when $0 \leq x+y < 1$ is $x = y = 0$.

Now suppose that $1 \leq x+y < 2$. Then the original equation can be written as $x+y-1 = xy$, which simplifies to $(x-1)(y-1) = 0$. Thus we must have $x-1 = 0$ or $y-1 = 0$, both of which are impossible as we are assuming that $0 \leq x < 1$ and $0 \leq y < 1$. Thus there are no solutions in this case.

Hence the only solution (x,y) satisfying the additional condition $0 \leq x < 1$ and $0 \leq y < 1$ is $x = y = 0$. Since we observed that (x,y) is a solution if and only if $(x+n, y+m)$ is a solution for all integers m and n, we conclude that the whole solution set is those pairs (x,y) so that both x and y are integers.

Solution 2. First we recall some standard rules for divisibility. Let $N = \overline{a_1 \ldots a_n}$ denote a n-digit number (written as usually in base 10, i.e. $a_i \in \{0, 1, \ldots, 9\}$ and $N = a_n + 10a_{n-1} + \cdots + 10^{n-1}a_1$). Then N is divisible by 3 if and only if the sum of its digits, i.e. $a_1 + a_2 + \cdots + a_n$ is divisible by 3. The same statement also holds when 3 is replaced with 9, but we don't need it in this problem. In addition, N is divisible by 11 if and only if the alternating sum of its digits $a_1 - a_2 + a_3 - a_4 + \cdots + (-1)^{n-1} a_n$ is divisible by 11.

Note that $132 = 3 \cdot 4 \cdot 11$. Since 3, 4 and 11 are pairwise coprime (do not have any common factors apart from 1), an integer N is divisible by 132 if and only if N is divisible by 3, N is divisible by 4 and N is divisible by 11. Note that one has to be a bit careful here: even though say $24 = 4 \cdot 6$, an integer N can be divisible by 4 and 6 without being divisible by 24 (say, 12) - but of course here the problem is that 4 and 6 share 2 as a common factor.

Let $N = \overline{a_1 \ldots a_n}$ denote the least such number with $a_i \in \{2, 3\}$ for every i. By checking small cases, it is not too hard to conclude that $N = 23232$ satisfies the conditions (indeed, it is clearly divisible by 4, the sum of digits is 12 which is divisible by 3 and the alternating sum of digits is 0 which is divisible by 11). Thus we may assume that the smallest N has at most 5 digits.

Let A denote the sum of digits with odd index (those of a_1, a_3, a_5, depending on how many digits N has) and let B denote the sum of digits with even index (a_2 and a_4). Then we know that 3 must divide $A+B$ and 11 must divide $A-B$. Since $0 \leq A \leq 3 \cdot 3 = 9$ and

$0 \leq B \leq 2 \cdot 3 = 6$, we conclude that $-6 \leq A - B \leq 9$. Since $A - B$ is divisible by 11, the only possibility is to have $A - B = 0$, i.e. $A = B$. But then $A + B = 2A$, and since 2 and 3 do not have any common factors apart from 1, A must be divisible by 3 and similarly B must be divisible by 3.

If N had at most 4 digits, then both A and B will be sums of at most 2 digits that are 2 or 3. The divisibility of such a sum clearly depends only on the number of 2's in it: the number of 2's needs to be divisible by 3. But since both sums contain only at most 2 digits, the only way such a situation can be achieved is by taking no 2's at all to either sets. This in turn implies that all digits are $3's$, whence N is clearly odd integer and thus not divisible by 4.

Now suppose N has 5 digits. Since B contains only 2 digits, by the previous argument both of them must be 3's. Now A contains 3 digits whose sum is divisible by 3, and again not all of them can be 3's. Thus at least one of them must be 2, and hence at least 3 of them must be 2's, whence all of them must be 2's. Hence we end up with the number $N = 23232$, which we know to satisfy these properties.

Finally, observe the simple fact that if N has at least 6 digits, then regardless of their choices, we must have $N > 23232$, thus proving that 23232 is the smallest such integer.

Solution 3. Apllying the law of sines to the triangle ABC implies that

$$\frac{BC}{\sin(\angle BAC)} = \frac{AB}{\sin(\angle ACB)}. \tag{1}$$

Since $\angle EBA = 90°$, we have

$$AB = \cos(\angle BAC) \cdot AE. \tag{2}$$

Substituting the value of AB from (2) into (1), and using the fact that $AE = 2 \cdot BC$, we obtain

$$\frac{BC}{\sin(\angle BAC)} = \frac{AB}{\sin(\angle ACB)} = 2\frac{\cos(\angle BAC)\, BC}{\sin(\angle ACB)}$$

which simplifies to $\sin(\angle ACB) = 2\cos(\angle BAC) \cdot \sin(\angle BAC) = \sin(2\angle BAC)$. Since both of these angles are between 0° and 180°, it follows that $\angle ACB = 2\angle BAC$ or $\angle ACB + 2\angle BAC = 180°$. Our aim is to rule out the second option.

Suppose we have $\angle ACB + 2\angle BAC = 180°$. Since the sum of the angles of triangle is 180°, it follows that $\angle CBA = \angle BAC$ by considering the triangle ABC. Thus we must have $BC = AC$, which in turn implies that $BC = AC > AE = 2 \cdot BC$. This is clearly a contradiction, so we must have $\angle ACB = 2\angle BAC$. Thus the diagonal AC divides the angle $\angle ACB$ and hence also the angle $\angle BDA$ in $2:1$ ratio.