Problem sheet 2

Problem 1. Let n be a number with digits $a_{m-1} \dots a_0$ - that is, we have $n = \sum_{j=0}^{m-1} a_j 10^j$. Prove that

1. 3 divides n if and only if 3 divides $\sum_{j=0}^{m-1} a_j$.
2. 9 divides n if and only if 9 divides $\sum_{j=0}^{m-1} a_j$.
3. 11 divides n if and only if 11 divides $a_0 - a_1 + a_2 \dots + (-1)^{m-1} a_{m-1}$.

Problem 2. Find all positive integers n for which $(n-1)!$ is divisible with n (here $n! = n \cdot (n-1) \cdot \dots \cdot 2 \cdot 1$).

Problem 3. When 4444^{4444} is written in decimal notation, let A be the sum of its digits. Let B be the sum of the digits of A. Find the sum of the digits of B.

Problem 4.

1. Find all integer solutions of the equation $x^2 = 2023 + y^2$.
2. Find all integer solutions of the equation $x^2 = 2022 + y^2$.

Problem 5. Let $a < b$ be positive integers. Prove that every block of b consecutive integers contains two distinct integers x and y so that $ab | xy$.

Problem 6. Prove that there does not exist positive integers x, y and z satisfying $x^2 + 3y^2 = 2z^2$.

Homework problems

You may submit your written solutions until the next meeting (7 November) in person, or by email (eero.raty@umu.se).

Problem 1. Let m be a given positive integer. Prove that there exists a positive integer n (depending on m) so that none of the numbers $n+1, \ldots, n+m$ is a prime number.

Problem 2. Determine all pairs of positive integers (x, y) satisfying $(xy-7)^2 = x^2 + y^2$.

Problem 3. 25 people decide to form some number of smaller clubs. Each such smaller club contains exactly 5 members, and any two smaller clubs have at most 1 common member. Prove that the maximum number of smaller clubs that can be formed is 30, and construct an example of 30 such clubs satisfying the required properties.