# Problem sheet 1

**Problem 1.** Using three distinct digits, we create the smallest and largest 3-digit number possible, and add them together. If the result is 1655, what are the three digits?

**Problem 2.** What is the maximal positive integer $n$ such that $n + 10$ divides $n^2 + 2023$?

**Problem 3.** In each field of a $3 \times 3$ table, we write one of the numbers $0, 1, 2$. We add the numbers in each row and each column. Is it possible that the resulting 6 numbers are all different?

**Problem 4.** Let $P$ be an interior point of the rectangle $ABCD$. Assuming $AP = 4, BP = 6$, and $DP = 9$, determine the length of $CP$.

# Homework problems

You may submit your written solutions until the next meeting (17 October) in person, or by email (istvan.tomon@umu.se).

**Problem 1.** Find all real solutions $x, y$ of the equation

$$\{x + y\} = \{x\} \cdot \{y\},$$

where $\{a\}$ is the *fractional part (also known as the decimal part) of a*. It is defined as $\{a\} = a - \lfloor a \rfloor$, where $\lfloor a \rfloor$ is the largest integer not larger than $a$.

*(7 points)*

**Problem 2.** Determine the smallest positive integer whose every digit is 2 or 3, and is divisible by 132.

*(7 points)*

**Problem 3.** Given a parallelogram $ABCD$, draw the line perpendicular to the side $AB$ from $B$. This line intersects the $AC$ diagonal in the interior point $E$. Assuming $AE = 2 \cdot BC$, in what ratio does $AC$ divide the angle $\angle BAD$?

*(7 points)*