

HOMWORK PROBLEMS 3

- (1) A point  $P$  is selected uniformly at random in a square  $ABCD$ . What is the probability that the angle  $\angle APB$  is acute?
- (2) In the parallelogram  $ABCD$ , the point  $E$  lies on  $AD$ , and the point  $F$  lies on  $BC$ , such that  $|ED| = |BF|$ . The point  $K$  lies on  $CD$ . Let  $P$  be the point of intersection between  $EF$  and  $AK$ , and let  $Q$  be the point of intersection between  $EF$  and  $KB$ .

Show that  $|\triangle APE| + |\triangle BQF| = |\triangle PKQ|$ .

- (3) The angle at  $C$  in  $\triangle ABC$  is  $45^\circ$ , and the angle at  $A$  is acute. The lines  $BD$  and  $CE$  are heights in the triangle, and  $E \neq B$ . The point  $F$  lies on  $AB$ , and  $DF$  is perpendicular to  $AB$ .

Show that if  $|CE| + |BE| = 2|DF|$ , then the angle at  $A$  is less than  $45^\circ$ , and conversely, if the angle at  $A$  is less than  $45^\circ$ , then  $|CE| + |BE| = 2|DF|$ .

You may submit your written solutions at the next meeting (November 29) in person, or by e-mail ([lars-daniel.ohman@umu.se](mailto:lars-daniel.ohman@umu.se)).