Midwinter meeting in discrete probability
January 15-16, 2025, Umeå

All talks are in room MC413 in the MIT-building.

Each speaker has either a 45 or a 40 minute slot which covers both the talk and questions.

Wednesday

9.15-10.00 Victor Falgas-Ravry On the geometry of random right-angled Coxeter groups

The right-angled Coxeter group (or RACG) $W = W(\Gamma)$ with presentation graph $\Gamma = (V, E)$ is the group with generators V and relations $aa = 1$ and $ab = ba$ for all $a \in V$ and $ab \in E$. By taking Γ to be an instance of the Erdős–Rényi random graph model on n vertices with edge probability $p = p(n)$, one can generate a model for a random RACG. In recent years, geometric group theorists have investigated this model with a particular emphasis on its typical geometric properties. In this talk I will explain how one can obtain a rough threshold for relative hyperbolicity in this model and discuss the related problem of square percolation in random graphs.

Joint work with Jason Berhstock and Altar Çiçeksiz.

10.00-10.30 Coffee

10.30-11.15 Timo Vilkas The averaging process on infinite graphs

We consider the averaging process on an infinite connected graph with bounded degree and iid starting values. Assuming that the law of the initial value has a finite second moment, we show that the values at all vertices converge in L^2 to the corresponding first moment (which is constant in time).

This is joint work with N. Gantert.

11.15-11.55 Stepan Vakhrushev Maximum number of extensions in the random graph

It is known that after an appropriate rescaling the maximum degree of the binomial random graph converges in distribution to a Gumbel random variable. The same holds true for the maximum number of common neighbours of a k-vertex set, and for the maximum number of s-cliques sharing a single vertex.

Can these results be generalised to the maximum number of extensions of a k-vertex set for any given way of extending of a k-vertex set by an s-vertex set? In our we generalise the above mentioned results to a class of "symmetric extensions" and show that the limit distribution is not necessarily from the Gumbel family.

Lunch 12.00-13.15

13.15-14.00 Klas Markström Negative correlations and the van den Berg-Kesten inequality

In this talk I will discuss the van den Berg-Kesten inequality, stating that $P(A\Box B) \leq P(A)P(B)$, where $A\Box B$ means that both events occur but on different subsets of the underlying random variables. The original inequality was proved for the usual product measure on $\{0,1\}^n$ and was much later also proved for k-out of-n measures. We have shown that various other measures satisfy the inequality and relate it to various other negative correlation properties like negative association and strongly Rayleigh measures.

This is based on joint work with Victor Falgas-Ravry.

14.00–14.45 Tobias Johnson The density conjecture for activated random walk

Physicists Bak, Tang, and Wiesenfeld in the 1980s proposed "self-organized criticality" as an explanation for why systems in nature with no obvious phase transition can exhibit self-similarity and power-law tails reminiscent of statistical mechanics systems at criticality. Based on simulations, they and others proposed that simple mathematical models of sandpiles drive themselves to criticality, in various senses. These predictions have been quite difficult to confirm mathematically. We consider activated random walk (ARW), one of the sandpile models that seems to follow physicists' predictions. In dimension one, we prove the density conjecture: ARW on a finite interval with particles added in the middle and destroyed at the edges naturally drives itself to the critical density of ARW on the infinite line. This is the first rigorous proof of any sandpile model driving itself to a critical state.

Joint work with Chris Hoffman and Matt Junge.

14.45-15.15 Coffee

15.15-16.00 Fabian Burghart 1-balanced factors in random graphs

Let F be a graph on r vertices and let G be a graph on n vertices. An F-factor in G is a subgraph of G composed of n/r vertex-disjoint copies of F, if r divides n. For instance, a K_2-factor is simply a perfect matching. The study of threshold functions for F-factors in $G(n,p)$ goes back to Erdös and Rényi themselves; but for general F it was not until the 2008 breakthrough paper by Johansson, Kahn and Vu that the weak threshold for strictly 1-balanced F was established. More recently, Riordan and Heckel obtained sharp thresholds for $F = K_r$ and so-called nice graphs, using sophisticated coupling arguments that utilize Kahn's recent celebrated solution of Shamir's problem on hypergraph matchings. In this talk, we review these results, as well as the recent extension of the sharp thresholds to any strictly 1-balanced F, obtained in joint work with A. Heckel, M. Kaufmann, N. Müller and M. Pasch. In particular, this confirms the thirty year old conjecture by Rucínski that the sharp threshold for the emergence of an F-factor indeed coincides with the sharp threshold for the disappearance of the last vertices which are not contained in a copy of F.

This is based on joint work with Annika Heckel, Marc Kaufmann, Noela Müller, and Matija Pasch

18.00 Dinner at Tapas Bar Deli for registered participants

Thursday

9.15-10.00 István Tomon Around the log-rank conjecture

The log-rank conjecture, proposed by Lovasz and Saks, is one of the fundamental problems of communication complexity. In its combinatorial formulation, it asks for the size of the largest all-zero or all-one submatrix of a binary matrix of rank r. I will talk about recent questions and developments surrounding this conjecture.

Based on joint works with Zach Hunter, Aleksa Milojevic and Benny Sudakov.

10.00-10.30 Coffee

10.30-11.15 Carl Johan Casselgren Coloring graphs and hypergraphs from random lists

11.15-11.55 Vilhelm Agdur A fixed-parameter tractability result for the sparse mixed-sign k-part minimum cut problem

We consider the problem of finding a minimal edge cut on a weighted graph in which some weights may be negative. In the setting where we want to divide the graph into two parts, this is known to be fixed-parameter tractable with the vertex cover number of the subgraph induced by the negative-weight edges as parameter (McCormick, Rao, Rinaldi. 2003), but this result crucially relies on the tractability of computing two-terminal maximum flow.

We prove, using a recent FPT result for multi-terminal cut (Kim, Masařík. O, Pilipczuk, Sharma, Wahlström. 2024), that this problem is tractable with the maximum degree of the graph and bounds on the weights as additional parameters. We also suggest the possibility of using approximation results for multi-terminal cut to approximate this problem for dense graphs - this part is, however, still very much a work in progress.

Lunch 12.00-13.05

13.05-13.50 He Guo Prague dimension of random graphs

The Prague dimension of graphs was introduced by Nesetril, Pultr and Rodl in the 1970s. Proving a conjecture of Furedi and Kantor, we show that the Prague dimension of the binomial random graph is typically of order n/log n for constant edge-probabilities. The main new proof ingredient is a Pippenger-Spencer type edge-coloring result for random hypergraphs with large uniformities, i.e., edges of size O(log n).

Based on joint work with Kalen Patton and Lutz Warnke.

13.50-14.35 Eero Räty Large Cuts in Hypergraphs

A simple probabilistic argument shows that every r-uniform hypergraph with m edges contains an r-partite sub-hypergraph with at least $\frac{r!}{r^r}m$ edges. The celebrated result of Edwards states that in the case of graphs, that is $r = 2$, the resulting bound $m/2$ can be improved to $m/2 + \Omega(m^{1/2})$, and this is sharp. We prove that if $r \geq 3$, then there is an r-partite sub-hypergraph with at least $\frac{r!}{r^r}m + m^{3/5-o(1)}$ edges. Moreover, if the hypergraph is linear, this can be improved to $\frac{r!}{r^r}m + m^{3/4-o(1)}$, which is tight up to the $o(1)$ term. These improve results of Conlon, Fox, Kwan, and Sudakov. Our proof is based on a combination of probabilistic, combinatorial, and linear algebraic techniques, and semidefinite programming.

This is joint work with István Tomon.

14.35 Coffee