

Project title:

Mixed degree conditions in hypergraphs

Topics:

Extremal combinatorics, hypergraphs

Location:

Umeå, Sweden

Research group:

Discrete mathematics research group (see the links for more details)

Advisor:

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Head of department:

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Project outline:

An r -uniform hypergraph (or r -graph) is a pair $H = (V, E)$, where $V = V(H)$ is a set of vertices, and $E = E(H)$ is a collection of r -sets from V that form the edges of H . The *degree* $\deg(S)$ of a set S in H is the number of edges of H that contain S . The minimum of $\deg(S)$ over all subsets $S \subseteq V$ of size $|S| = \ell$ is known as the *minimum ℓ -degree* of H , and is denoted by $\delta_\ell(H)$.

A copy of the clique on t vertices $K_t^{(r)}$ in H is a t -set $X \subseteq V$ such that all $\binom{t}{r}$ possible r -sets from X are present as edges in H . If H contains no such copy, H is said to be $K_t^{(r)}$ -free. A classical problem in extremal hypergraph theory is: given integers $1 \leq \ell < r < t$, determine the maximum value of $\delta_\ell(H)$ over all $K_t^{(r)}$ -free r -graphs H on $n = |V(H)|$ vertices. The case $r = 2$ was resolved almost a century ago in a celebrated theorem of Turán [3].

For all $r \geq 3$, however, this problem — known as the *hypergraph Turán problem* — has remained stubbornly open in all cases, and is arguably one of the major open problems in discrete mathematics; see Keevash's survey [1] dedicated to it.

As part of an internship, I proposed to study a mixed-degree version of the hypergraph Turán problem. The focus would be on obtaining good lower bound constructions for this new family of problems, which is as yet unstudied, in order to cast light on the hypergraph Turán problem. Part of the work would build on a mountain of unpublished data on flag algebraic bounds computed by Markström, Sliacan and the advisor, and on a study of the treasure trove of constructions provided by Sidorenko in his 1995 survey [2].

References

- [1] Peter Keevash. Hypergraph Turán problems. *Surveys in combinatorics*, 392:83–140, 2011.
- [2] Alexander Sidorenko. What we know and what we do not know about Turán numbers. *Graphs and Combinatorics*, 11(2):179–199, 1995.
- [3] Paul Turán. On an extremal problem in graph theory. *Mat. Fiz. Lapok*, 48:436–452, 1941.

Objective:

The research internship is expected to lead to a publication suitable for a good research journal in extremal combinatorics.

The intern will begin by familiarising themselves with previous work before attacking a series of sub-problems suggested by the advisor. In addition to the advisor, the internship may involve collaboration with other group members, and the intern will be expected to take part in the life of the discrete mathematics research group and its weekly seminar.

Expected ability of the student:

A suitable intern must have some basic background in combinatorics and graph theory, a high level of mathematical ability and an enthusiasm for mathematical research. Previous exposure to extremal graph theory is desirable, but not required.

Life in Umeå:

In addition to the advisor, researchers in combinatorics at Umeå include István Tomon, Klas Markström, Maryam Sharifzadeh, Klara Stokes, Eero Rätty, Sabrina Lato and He Guo as well as half a dozen PhD and master’s students. The group has broad interests, and is friendly and highly international. There is a weekly seminar and the group enjoy frequent visits by guests researchers.

Umeå itself is a city in northern Sweden of around 130 000 inhabitants, of whom over 35 000 are students. It has a rich cultural life, with many pubs, cafés, restaurants, art galleries, concerts and festivals, which led to it being appointed European capital of culture jointly with Riga in 2014. The town itself is safe and well-run, with an extensive network of bicycle paths allowing its denizens to cycle all year round. It is surrounded by beautiful nature — from the Umeå river, which freezes over in winter and becomes criss-crossed with cross-country skiing tracks, to Nydala lake and the Gammlia forest within the city limits, with the sea and a number of natural reserves close by — and offers aurora sightings in Winter and endless days in Summer.

Other information:

The advisor is happy to supervise an internship in French, English or Swedish. Students interested in an internship are encouraged to contact the advisor by email (in French, English or Swedish) to discuss both the project and any questions they may have about life and combinatorics in Umeå.